DSP Supported Sweeping Spectrum Analysis

Laszlo Hars
Philips Research
Email: Laszlo.Hars@Philips.com

***Abstract*:** Sweeping spectrum analyzers can achieve several GHz measurement bandwidths at relatively low-cost. We discuss some design and implementation issues of DC mixing sweeping spectrum analyzers: resolution filter design, sweep speed and estimation of the power of 0 frequency mix products, etc.

Introduction

Sweeping spectrum analysis makes it possible to build inexpensive Spectrum Analyzers of several GHz measurement bandwidths. It employs a continuously tuned oscillator sweeping through the frequencies of the measurement band offset by a certain fixed frequency f_0. The input signal is mixed with this local oscillator and the mixing products around frequency f_0 get filtered out. The power of this signal corresponds to the power of the component of the input signal at the current frequency of the sweeping oscillator plus the filter frequency.

The filter frequency f_0 is often chosen to be very high to avoid the mirror frequency problem (frequencies other than the ones to be measured also mix into the filter band). High filter frequency allows easier elimination of the disturbing frequency components but it requires high precision electronics working usually at several GHz. Another alternative is to use an $f_0 = 0$ Hz filter frequency (low-pass filter), when there is no mirror frequency at all. In this case sweeping spectrum analysis requires the accurate measurement of the power of the mixed low-frequency signals.

Design issues

Unfortunately, there are inherent difficulties in measuring the power of 0 frequency (DC) mixing products: the random phase difference between the mixer and the input signal causes uncertainties. We propose an alternative method:

1. Remove the 0 frequency components with a sharp high-pass filter
2. Measure the power of the remaining very low but non-zero frequency components
3. Interpolate the results to DC.

The interpolation is essential, since the high-pass filter removes the DC components, causing a sharp notch at the center of the spectral peaks. The usual interpolation and pattern matching algorithms don't work very well. In the presence of noise they produce visible and disturbing artifacts on the spectrum display. We developed a better new technique, the "shift-min-add" algorithm:

1. Shift the spectrum graph along the frequency axis in either direction by about a quarter of the resolution bandwidth
2. Determine the lower envelope graph formed by the lowest points of these curves at each frequency. It has a sharp peak at the frequency of the input signal.
3. This curve will be added to the original display curve. Although the lower envelope curve is of distorted shape, it is well suited to filling the notch in the display curve.

Experiments have shown that this algorithm is superior to standard methods in producing no disturbing visual effects, yet giving accurate interpolation to DC. The notch filling curve shows peaks of heights proportional to the power of the corresponding signal components and the shape of these peaks does not change with frequency or amplitude.

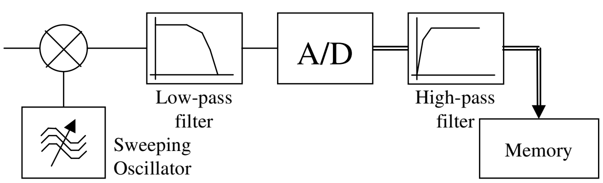
Another design issue is the tradeoff between the "reaction time", i.e. the impulse response of the resolution filter and the maximum sweep speed.

The stability and linearity of the sweeping oscillator is crucial for the overall measurement accuracy. Precise, stable and linearly sweeping oscillators are expensive to build. A much more economical solution is to employ less stable and less accurate oscillators and continuously measure their instantaneous frequency, e.g., by pulse counting. At the end of the sweep we check if it was of the correct speed. If not, adjust it and repeat the sweep. Having a sweep of the correct speed, render the measurement data according to the oscillator frequencies. DSP techniques are very efficient for this purpose, but due to lack of space we cannot go into detail here. (Recent developments in digital synthesized oscillators with analog frequency multipliers give a more accurate but also more expensive solution.)

DC-Mixing Spectrum Analyzers

The signal processing consists of analog/on-line and digital/off-line processing.

On-line signal processing

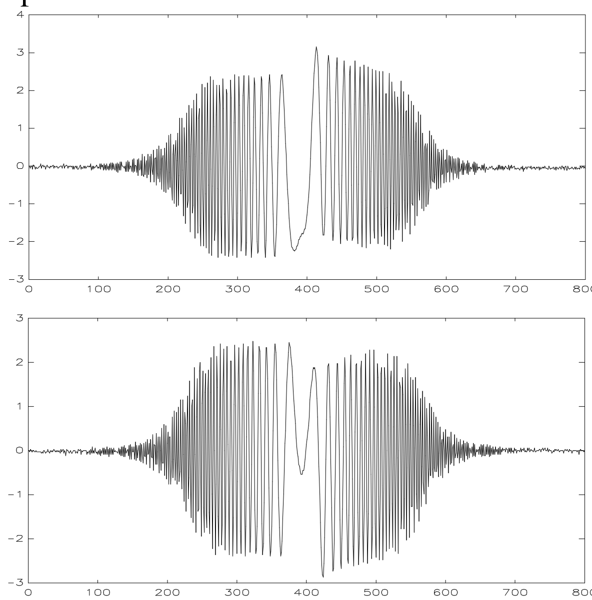


The signal to be measured gets mixed with the sine wave output of the local oscillator, which sweeps through the measurement band. A low pass filter removes the high frequency mixing products. Its cut-off frequency is half of the measurement bandwidth. Its design is discussed in the next section.

An Analog to Digital converter provides digital samples of the low frequency analog signal. The sampling rate must larger than the measurement bandwidth. The tradeoffs are presented below in the digital low-pass filter design discussion.

We are not concerned this point with the phase fidelity, so a sharp infinite impulse response (IIR) filter is applied to remove DC components. Implementing the IIR filter requires only a handful of DSP instructions, which does not add much to the data processing (wait, read, convert, store) needed anyway. This is why it is done on-line, before the data is stored for off-line processing.

We can use AC coupling in the analog signal path, eliminating temperature drift problems. Unfortunately the same input signal can give different voltage levels at the beat frequency. It can charge the coupling capacitors positive or negative, according to the phase of the local oscillator.



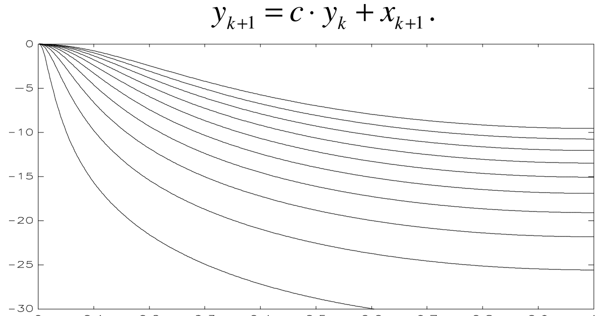
Oscilloscope displays of signals at different mixer phases

The ***low-pass exponential averaging filter*** is defined by

$$r = c \cdot r + x_{k+1},\ \ y_{k+1} = r\ ,$$

where r can be a DSP register. It can also be written as a conventional IIR filter definition:

$$y_{k+1} = c \cdot y_k + x_{k+1}\,.$$

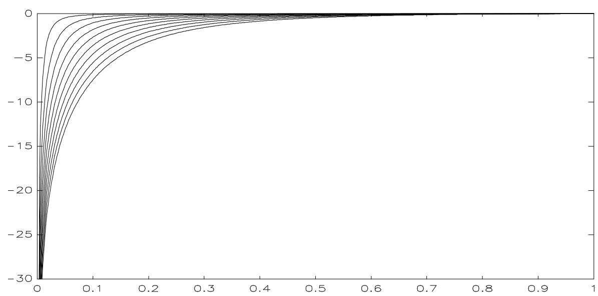


Frequency response of low-pass exponential averaging filters with c = 0.5, 0.55,…0.95

The ***high-pass exponential averaging filter*** is obtained from the previous one by subtracting the low-pass filtered signal from the original:

$$r = c \cdot r + x_{k+1}, \quad y_{k+1} = r - \frac{x_{k+1}}{1-c}$$

When r is a DSP register 4 instructions are enough to implement this filter.



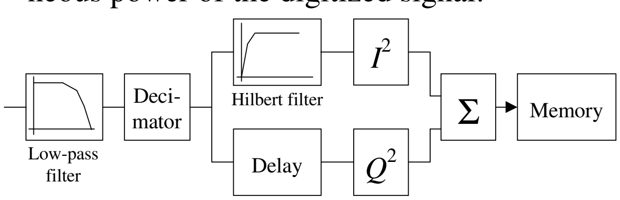
Frequency response of high-pass exponential averaging filters with c = 0.5, 0.55,…0.95

The definition in the usual IIR filter format is:

$$y_{k+1} = -\frac{c}{1-c} x_{k+1} + \frac{c}{1-c} x_k + c \cdot y_k \, .$$

Off-line, digital signal processing

(With a fast DSP these steps can be performed on-line, too – reducing the time between sweeps.) First we calculate the instantaneous power of the digitized signal:



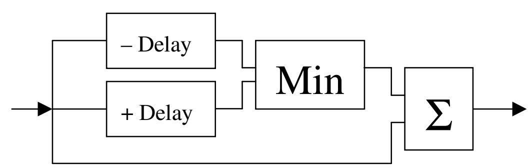
If we use an ADC, e.g. of 12 bit resolution, it produces a quantization noise of $2^{-11} \approx -66$ dB. This is too high, but with a digital ***low-pass filter*** the quantization noise can be reduced and the dynamic range of the measurement extended.

To improve the effective ADC resolution the sampling rate is chosen as high as possible but after the digital low-pass filter, we have to ***decimate*** to speed-up the subsequent signal processing. This means we trade speed for resolution. A single ADC is used for different tasks. In low frequency spans (slow frequency change) we average a lot of ADC samples to improve the dynamic range. This cannot be done in large spans without reducing the resolution bandwidth of the measurement, but there it is not needed. At large resolution bandwidths the received and inherent instrument noise is larger than the quantization noise of the ADC.

The task of the ***Hilbert filter*** is to shift phase by 90º. This cannot be done at 0 Hz frequency and if we design filters, which approach this limit better, the filters get longer. Fortunately we removed the low frequency components during the on-line processing, so the frequency limit can be the same as at the previous high-pass filter.

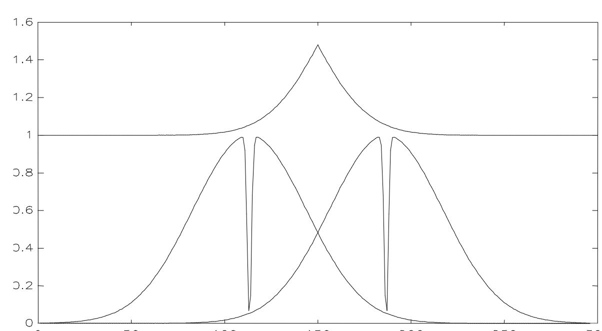
The instantaneous ***power*** is simply the sum of the squares of the 90º phase shifted samples. To match the samples both signal paths must have the same ***delay***, so the unfiltered samples are read from the memory with the appropriate offset.

Next the DC notch is removed:



Minimum of shifts

If we shift the spectrum graph with the DC notch a little to the left and to the right (the 2 bottom curves on the diagram) and take the minimum of the 2 waveforms, we get a smaller waveform which does not exhibit a notch at DC.



The disadvantages of this method are that the resulting waveform is of unusual shape, and being the result of a minimum operation, it is small and closer to the noise floor.

Sum with minimum of shifts

Although this is not good enough to display, it can be added (scaled by a normalizing factor) to the original envelope. This cancels out the DC notch and, because the minimum waveform is narrower, it does not significantly increase the resolution bandwidth.

The delay length is experimentally chosen, such that the displayed spectrum looks the best (smoothest top). It is approximately ¼ of the resolution bandwidth.

The final signal processing step is to fit the frequencies of the just-determined spectrum data to the measured sweeping oscillator frequencies. The standard data fitting routines work fine here (see [5]).

The Analog Low-pass filter

For the definition of the frequency response of an analog filter the Laplace transform $\mathcal{L}$ is needed. It is defined by:

$$\mathcal{L}\left(f\left(t\right)\right)=F\left(s\right)=\int_{0}^{\infty}e^{-st}f\left(t\right)dt\,.$$

Let $f(t)$ be the input voltage applied to the filter and let $g(t)$ be the output voltage. The frequency response corresponding to $h(t)$ is found from the relation:

$$\mathcal{L}\left(g\left(t\right)\right)=\mathcal{L}\left(h\left(t\right)\right)\cdot\mathcal{L}\left(f\left(t\right)\right).$$

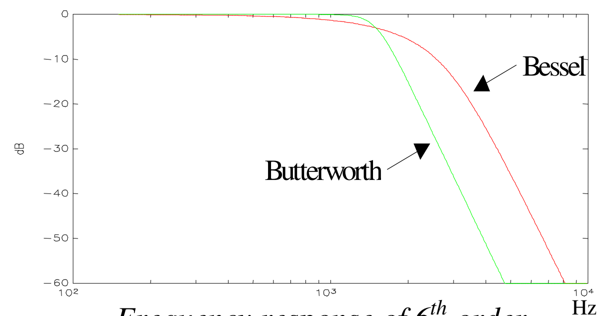
In case of 6th order low-pass Bessel or Butterworth filters with angular cut-off frequency $\omega = 2\pi f = 1$ Hz, the filter response is well known:

$$\mathcal{L}\left(h\left(t\right)\right)=H\left(s\right)=\frac{1}{1+c_1 s+\cdots+c_6 s^6}\,,$$

where the coefficients are the following

	C_1	C_2	C_3	C_4	C_5	C_6
Bessel	2.7034	3.3220	2.3948	1.0790	0.2917	0.0375
Butter-worth	3.8637	7.4640	9.1415	7.4640	3.8637	1

Substituting the imaginary circular frequency $2\pi f\cdot i$ for s, we get the frequency response of the filter, the magnitude of $H(f)$.



Frequency response of 6^{th} order 1.5 KHz low-pass filters

We need to determine the output of a filter when a sweeping frequency input is applied to it. Unfortunately, the Laplace transform of the sweeping signal cannot be expressed in closed form, so we need numerical approximations. Also, numerical approximations are needed for the inverse Laplace transform to get the output signal in the time domain.

Take the inverse Laplace transform of the equation defining the filter response (the Laplace transform of the convolution of two functions is the product of the transforms of the functions):

$$g\left(t\right)=h\left(t\right)*f\left(t\right)=\int_{0}^{t}h\left(t-u\right)\cdot f\left(u\right)du\,.$$

To evaluate this the inverse Laplace transform $h(t)$ of the filter transfer function $H(s)$ is needed. The integral is easy to calculate numerically.

The roots of the response functions above are all different and neither a real nor an imaginary part is 0. Since the coefficients are real numbers, the roots form complex conjugate pairs. In this case the transfer functions can be written as sums of partial fractions in the following form:

$$\sum_{k=1}^{3}\left(\frac{\alpha_k}{s-\beta_k}+\frac{\bar{\alpha}_k}{s-\bar{\beta}_k}\right),\ \alpha_k = p_k + i\cdot q_k\ ,\ \beta_k = u_k + i\cdot v_k$$

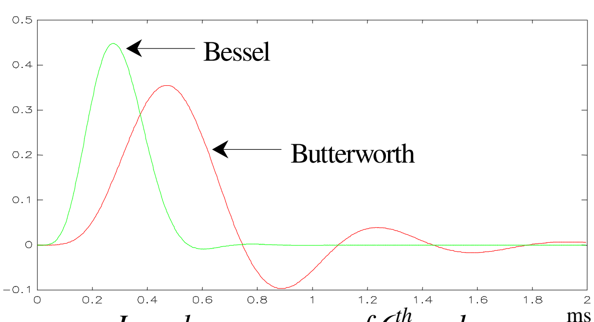
(p_k, q_k, u_k and v_k are reals.) Since the Laplace transform and its inverse is linear, $h(t)$ can be expressed as the sum of the inverse Laplace transforms of functions $\alpha/(s\text{-}\beta)$ that is, as the sum of $\alpha\cdot e^{\beta t}$, with α , β complex. With the above coefficients we get

$$h(t)=\sum_{k=1}^{3} 2e^{u_k t}\left(p_k\cos(v_k t)-q_k\sin(v_k t)\right).$$

A little calculation gives the following table of coefficients at $\omega_0 = 2\pi f_0$ cut-off frequency

	k	p_k	q_k	u_k/ω_0	v_k/ω_0
Bessel	1	1.1721	-0.0757	-0.9309	1.6619
	2	-5.2267	4.6983	-1.3817	0.9715
	3	4.0546	-14.5813	-1.5715	0.3209
Butter-worth	1	1.3193	2.2860	-0.9660	-0.2587
	2	0.2041	-0.3535	-0.2588	-0.9659
	3	-1.5234	0.0000	-0.7071	0.7071

If the input is $f(t) = \delta(t)$, the Dirac delta function, the infinitely narrow unit-power impulse at time 0, the output is the above $h(t)$. (This is why it is also called the impulse response of the filters.)



Impulse response of 6th order 1.5 KHz low-pass filters

Digital simulations

In order to determine the minimum useful sweep time we used digital simulation of the analog low-pass filters, since no analytical solution is known. The method we used is the *impulse invariant transformation.*

The sweeping input signal

If the input signal of the spectrum analyser is a sine wave of frequency equal to the center frequency of the measured band, after mixing with the sweeping local oscillator we get a sinusoidal signal $\sin(2\pi f(t))$ with frequency linearly increasing from $-f_0$ to f_0, as time goes from 0 to t_0, (t_0 is the sweep time; $2f_0$ the frequency span of the measurement): $f'(t) = \gamma t+\delta$, $f'(0) = -f_0$, $f'(t_0) = f_0$. This gives (with arbitrary phase φ)

$$f(t)=\frac{f_0}{t_0}t^2 - f_0 t+\varphi$$

This will be used for the digital simulation with $\varphi = 0$ and ¼.

The convolution integral

The usual quadrature procedures can be used to calculate the convolution integrals. However, for the digital simulation we need thousands of time points; this means thousands of slow integral evaluations. Therefore, we have approximated the integral with the sum of the function values of equidistant arguments. The convolution integrals are replaced by fast convolutions of sequences, calculated by FFT. When the number of points is $2^{16} = 65536$ the sum approximates the integral within a few percent error.

Let us fix n, the number of samples used in the digital simulation. Let $t_k = t_0\cdot k/n$, where the simulated time interval goes from 0 to t_0, $f_m = f(t_m)$, $g_m = g(t_m)$ and $h_m = h(t_m)$. We have

$$g_m=\int_0^{t_m} h(t_m-u)\cdot f(u)\,du \approx \frac{1}{t_0}\sum_{k=0}^{m-1} h(t_m-t_k)\cdot f(t_k)=\frac{1}{t_0}\sum_{k=0}^{m-1} h_{m-k}f_k$$

i.e., the continuous convolution is approximated by the (acyclic) discrete one. If we extend the sequences f_m, h_m with n zeros, we can use cyclic sequences, and apply the cyclic convolution, since the extra terms are all 0:

$$g_m \approx \frac{1}{t_0}\sum_{k=0}^{2n-1} h_{m-k} f_k, \quad 0 \le m < n.$$

This doubling of the size of the discrete convolution can be avoided now, because the sequence h_m rapidly decreases, and replacing it with the cyclic (repeated on both sides) version causes no significant error within the simulation time interval.

We ended up having to calculate

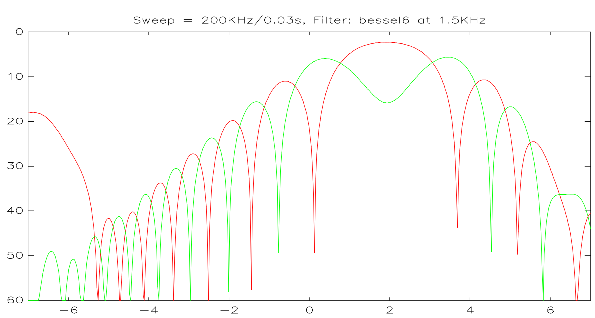
$$g_m \approx \frac{1}{t_0}\sum_{k=0}^{n-1} h_{m-k} f_k, \quad 0.1n \le m < 0.9n$$

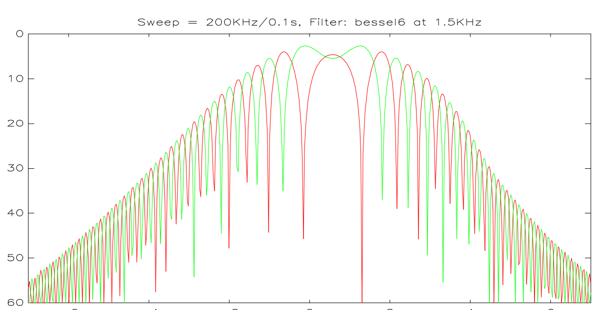
for our special cases. The fastest way to compute the sequence g_m is with element-wise multiplication "·":

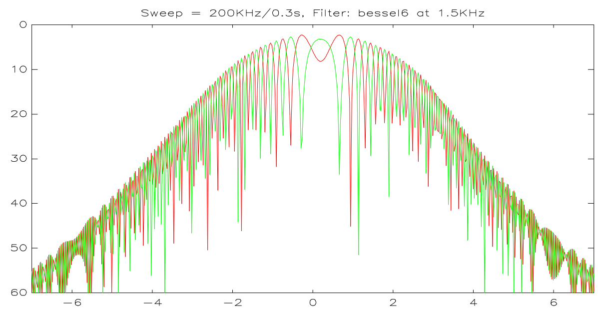
$$\{g_m\} = \mathrm{F}^{-1}\left(\mathrm{F}(\{h_m\})\right)\cdot\left(\mathrm{F}(\{f_m\})\right),$$

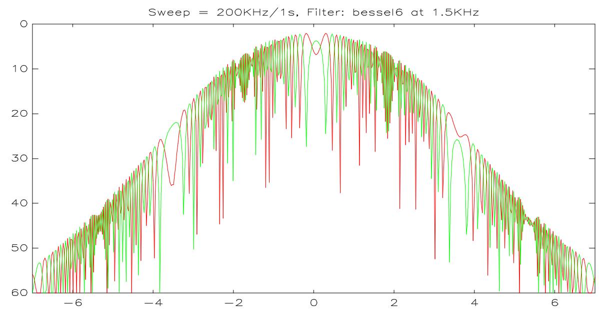
where F, F^{-1} denote the fast Fourier transform and its inverse, respectively.

Two different input signals are used: the sweeping sine wave and its replica shifted by 90º. This way we see better the behavior of the filters around 0 frequency.



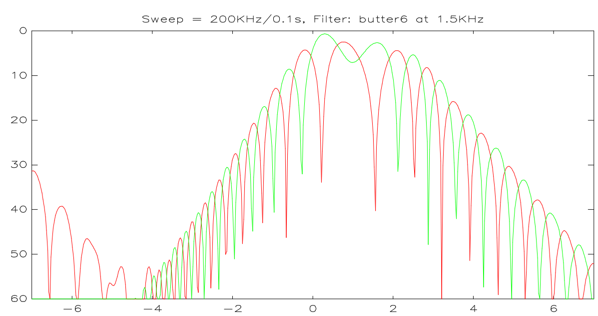


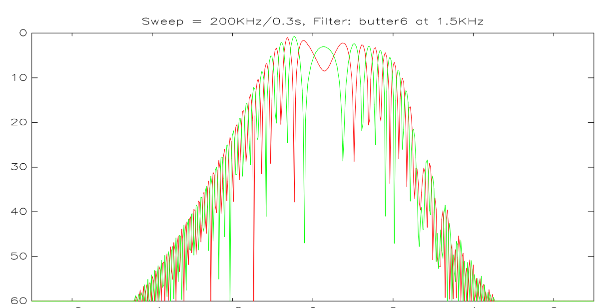


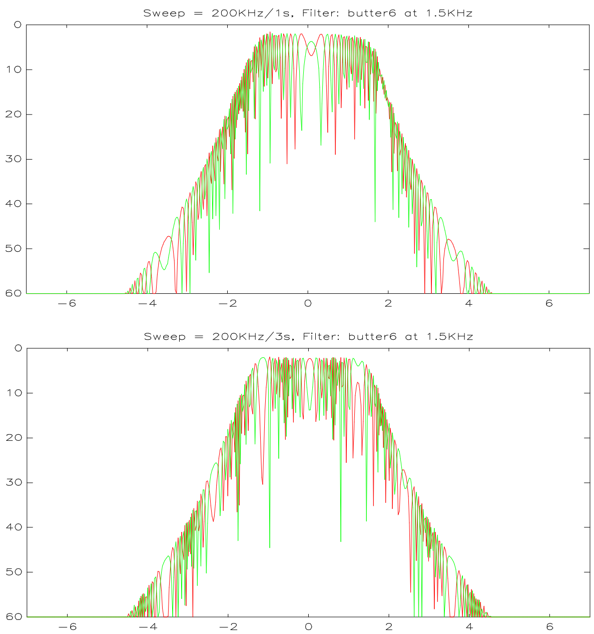


Sweeps with Bessel resolution filter

The response curve of a 1.5 KHz 6th order Bessel filter is almost acceptable at 100 ms sweep time: the output envelope shows only little asymmetry. At faster sweeps the main lobe is asymmetric; there is a significant delay of the peak from 0 frequency. These kinds of errors are not present when the sweep time is at least 300 ms.







Sweeps with Butterworth resolution filter

The response curve of the 1.5 KHz 6th order Butterworth filter is acceptable at 1s sweep time. There are no errors visible when the sweep time is at least 3s.

<u>Maximum sweep speed</u>

If we keep Δf (the frequency span) fixed, the graph of the filter response vs. time remains unchanged when $t_{sweep} \times f_{RBW} = constant$. ($t_{sweep}$ is the sweep time, f_{RBW} is the resolution bandwidth, the cut-off frequency of the filter.) Similarly if we keep t_{sweep} fixed, the graph of the filter response vs. time remains unchanged when $\Delta f \,/\, f_{RBW} = constant$. From these two equations we get

$$t_{sweep} \geq k \cdot \frac{\Delta f}{f_{RBW}^2}.$$

Here the constant k only depends on the filter. In case of the 6th order Bessel filter, the minimum feasible sweep time is around 180 ms, which gives $k = 2$. In case of the 6th order Butterworth filter, a little larger than 1s sweep time is the minimum, giving $k = 12$. The following table summarizes the minimum allowable sweep times using Bessel filters:

Span	Filter Cut-off Frequency	Min Sweep Time
200 KHz	1.5 KHz	178 ms
500 KHz	1.5 KHz	444 ms
1 MHz	5 KHz	80 ms
2 MHz	5 KHz	160 ms
5 MHz	15 KHz	44 ms
10 MHz	15 KHz	89 ms

Sweep times with Butterworth filters are 6 times longer.

Of course one can use different order filters than the standard 6th order. However, lower order filters do not give enough resolution off the center frequency, while higher order filters are slower, requiring longer sweep time.

There is an analog and a digital low-pass filter in the signal path. The lesser of their bandwidths determines the resolution of the spectrum analyzer. It is tempting to use only one analog filter of the largest necessary bandwidth followed by a digital filter after the ADC. With appropriate coefficients any desired resolution bandwidth can be achieved.

The drawback of this approach becomes apparent when complex waveforms are measured. At large analog bandwidths large signal power reaches the ADC. Because of the large corresponding peak amplitude the signal has to be attenuated more than with narrower analog filters to avoid overflow. This reduces the dynamic range of the instrument.

Consequently, a better but more expensive solution is to have a whole set of analog filters. The one having larger but closest-to-required bandwidth is switched on and the digital filter has to reduce the overall bandwidth by only a little.

Bibliography

R. Goodwin: *Speeding up Spectrum Analyzer Spurious Testing*, Microwaves & RF, August 1987, pp. 107-114.